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New relationships connecting a class of fractal objects and fractional integrals in space

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Abstract

Many specialists working in the field of the fractional calculus and its applications simply replace the integer differentiation and integration operators by their non-integer generalizations and do not give any serious justifications for this replacement. What kind of "Physics" lies in this mathematical replacement? Is it possible to justify this replacement or not for the given type of fractal and find the proper physical meaning? These or other similar questions are not discussed properly in the current papers related to this subject. In this paper new approach that relates to the procedure of the averaging of smooth functions on a fractal set with fractional integrals is suggested. This approach contains the previous one as a partial case and gives new solutions when the microscopic function entering into the structural-factor does not have finite value at $N \gg 1$ (N is number of self-similar objects). The approach was tested on the spatial Cantor set having M bars with different symmetry. There are cases when the averaging procedure leads to the power-law exponent that does not coincide with the fractal dimension of the self-similar object averaged. These new results will help researches to understand more clearly the meaning of the fractional integral. The limits of applicability of this approach and class of fractal are specified.

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Keywords

averaging of smooth functions on spatial fractal sets, Cantor set, Cantor set: fractal object, fractal object, self-similar object, spatial fractional integral